Expected Commodity Returns and Pricing Models

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Abstract

Stochastic models of commodity prices have evolved considerably in terms of their structure and the number and interpretation of the state variables that model the underlying risk. Using multiple factors, different specifications and modern estimation techniques, these models have gained wide acceptance because of their success in accurately fitting the observed commodity futures' term structures and their dynamics. It is not well emphasized however that these models, in addition to providing the risk neutral distribution of future spot prices, also provide their true distribution. While the parameters of the risk neutral distribution are estimated more precisely and are usually statistically significant, some of the parameters of the true distribution are typically measured with large errors and are statistically insignificant. In this paper we argue that to increase the reliability of commodity pricing models, and therefore their use by practitioners, some of their parameters –in particular the risk premiums parameters- should be obtained from other sources and we show that this can be done without losing any precision in the pricing of futures contracts. We show how the risk premium parameters can be obtained from estimations of expected futures returns and provide alternative procedures for estimating these expected futures returns.

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1. Introduction

Stochastic models of commodity prices have evolved considerably during recent years in terms of their structure and the number and interpretation of the state variables that model the underlying risk [Gibson and Schwartz (1990), Schwartz (1997), Schwartz and Smith (2000), Cortazar and Schwartz (2003), Cortazar and Naranjo (2006)]. Using multiple factors, different specifications and modern estimation techniques, these models have gained wide acceptance because of their success in accurately fitting the observed commodity futures' term structures and their dynamics.

Most of the commodity price models are calibrated using only futures panel data[1]. They assume that there are no-arbitrage opportunities in trading within these contracts and that the underlying process for commodity prices may be derived using only futures prices. These models provide the risk adjusted distribution of future spot commodity prices that, under the risk neutral framework, may be used to price all types of commodity derivatives and real options.

It is not well emphasized however that these models, in addition to providing the risk neutral distribution of future spot prices, also provide their true distribution. Even though the commodity price distribution under the (true) physical measure is unnecessary for valuation purposes, it is still important for at least two reasons. First, the true distribution is useful for non-valuation purposes, such as risk management (i.e. calculations of *Value at Risk*). Second, many practitioners still do not use the risk neutral approach for valuing natural resource investments, but instead use commodity price forecasts and then discount the expected cash flows generated with those forecasts at the weighted average cost of capital[2].

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[1] Some commodity models use also additional information, including Schwartz (1997) and Casassus and Collin-Dufresne (2005), which consider bond prices and Geman and Nguyen (2005) that incorporate inventory data. Also Cortazar et al. (2008) and Cortazar and Eterovic (2010) formulate multi-commodity models which use prices from one commodity to estimate the dynamics of another, and Trolle and Schwartz (2009) use commodity option prices to calibrate an unspanned stochastic volatility model.

[2] The International Valuation Standards Council (IVSC) released the discussion paper *Valuation in the Extractive Industries* in July 2012. Different questions about valuation methodologies where stated in this paper which industry participants were invited to answer. These answers where published and can be accessed at http://www.ivsc.org/comments/extractive-industries-discussion-paper. Respondents include the Valuation Standards Committee of the SME, The VALMIN Committee, the CIMVal committee and the American Appraisal Associates among others. Most of the respondents stated that their main method of valuation was a discounted cash flow analysis (DCF) using various methods of price forecasting. For the discount factor the most widely used method was a weighted average cost of capital (WACC) based on the Capital Asset Pricing Model (CAPM).

Thus, not only the risk adjusted process for valuing derivatives is of interest for users of commodity models, but also expected spot prices and their dynamics under the physical measure.

It is well known that expected future spot and futures prices differ only on the risk premiums, since futures prices are expected spot prices under the risk neutral measure. And here lies the problem: while the parameters of the risk neutral distribution are estimated more precisely and are usually statistically significant, some of the parameters of the true distribution are typically measured with large errors and are statistically insignificant [Schwartz (1997), Cortazar and Naranjo (2006)]. Thus, if these risk premiums are not well estimated, even though futures prices may not be affected, expected spot prices under the physical (true) measure will be[3]. So, when these models are used to infer anything about the true distribution of spot prices (e.g. NPV or risk management) they become very unreliable.

In this paper we argue that to increase the reliability of commodity pricing models, and therefore their use by practitioners, some of their parameters –in particular the risk premiums parameters- should be obtained from other sources and we show that this can be done without losing any precision in the pricing of futures contracts. We show how the risk premium parameters can be obtained from estimations of expected futures returns and provide alternative procedures for estimating these expected futures returns.

The remaining of the paper is as follows: Section 2 illustrates the nature of the problem using as an example the Schwartz-Smith (2000) commodity pricing model, and Section 3 shows how to estimate expected futures returns in this model. Section 4 describes alternative ways of estimating expected future returns and Section 5 presents empirical results of implementing our methodology for Copper and Oil futures. Finally, Section 6 concludes.

[3] In an independent work, Heath (2013) also finds that a futures panel is well suited for estimating the cost of carry, relevant for futures prices, but not the risk premiums, required for expected spot prices.

2. An Example

To illustrate more precisely the nature of the problem we use the two-factor Schwartz-Smith (2000) commodity model which has been widely used by academics and practitioners[4].

The first state variable of this model (ξ_t), represents the long term equilibrium (log) price level, while the second state variable (χ_t), represents short term mean-reverting variations in (log) prices. The log spot price (S_t) is then defined in Equation (1) as the sum of the state variables. Equations (2) and (3) present the stochastic processes (under the physical measure) followed by the state variables, where μ_ξ, κ, σ_ξ and σ_χ are parameters of the model.

$$\ln(S_t) = \chi_t + \xi_t \tag{1}$$

$$d\xi_t = \mu_\xi dt + \sigma_\xi dz_\xi \tag{2}$$

$$d\chi_t = -\kappa\chi_t dt + \sigma_\chi dz_\chi \tag{3}$$

Furthermore, dz_ξ and dz_χ are correlated Brownian motions with correlation $\rho_{\chi\xi}$, such that:

$$dz_\chi dz_\xi = \rho_{\chi\xi} dt \tag{4}$$

Equations (5) to (7) present the stochastic processes followed by the state variables under the risk neutral measure, where λ_χ and λ_ξ are the risk premiums which are assumed to be constant.

$$d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dz_\xi^Q \tag{5}$$

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dz_\chi^Q \tag{6}$$

$$dz_\chi^Q dz_\xi^Q = \rho_{\chi\xi} dt \tag{7}$$

[44] We came across this problem in conversations with a very large mining company which was using this model to value their real options.

Some relevant results of the Schwartz and Smith (2000) model are the expected value at time t of the state variables at time T, their covariance matrix and the expected value of the spot price. These are presented in Equations (8) through (10), respectively.

$$E_t\left(\begin{bmatrix}\chi_T\\ \xi_T\end{bmatrix}\right) = \begin{bmatrix}e^{-\kappa(T-t)}\chi_t\\ \mu_\xi(T-t)\end{bmatrix} \tag{8}$$

$$Cov_t\left(\begin{bmatrix}\chi_T\\ \xi_T\end{bmatrix}\right) = \begin{bmatrix}\left(1-e^{-2\kappa(T-t)}\right)\frac{\sigma_\chi^2}{2\kappa} & \left(1-e^{-\kappa(T-t)}\right)\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\\ \left(1-e^{-\kappa(T-t)}\right)\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa} & \sigma_\xi^2(T-t)\end{bmatrix} \tag{9}$$

$$E_t(S_T) = \exp[e^{-\kappa(T-t)}\chi_t + \xi_t + A'(T-t)]$$

$$\begin{aligned}A'(T-t) = \mu_\xi(T-t) &+ \frac{1}{2}\left(\left(1-e^{-2\kappa(T-t)}\right)\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2(T-t) + 2(1\right.\\ &\left.- e^{-\kappa(T-t)})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)\end{aligned} \tag{10}$$

Furthermore the price of a futures contract at time t that matures at time T ($F_{T,t}$) is given by the expected spot price under the risk neutral measure ($E_t{}^Q[S_T]$). Therefore the futures price is:

$$F_{T,t} = \exp[e^{-\kappa(T-t)}\chi_t + \xi_t + A(T-t)] \tag{11}$$

$$\begin{aligned}A(T-t) = (\mu_\xi - \lambda_\xi)(T-t) &- \left(1-e^{-\kappa(T-t)}\right)\frac{\lambda_\chi}{\kappa}\\ &+ \frac{1}{2}\left(\left(1-e^{-2\kappa(T-t)}\right)\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2(T-t) + 2(1-e^{-\kappa(T-t)})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)\end{aligned}$$

Notice that the only difference between Equations (10) and (11) are the risk premium parameters (lambdas). If the risk premiums were zero, then futures prices would coincide with expected spot prices.

Consider now an extreme example of the issue we want to illustrate. Between January 2009 and December 2012 COMEX copper prices increased by almost 160% (from 1.40 to 3.65 US$ per pound). Table 1 presents the model parameters estimated using a Kalman[5] filter using all futures price data from this period.

Parameter	Kalman filer parameters		
	Estimate	S.D	t-Test
κ	0.111	0.012	9.513
σ_X	0.910	0.069	13.180
λ_X	0.036	0.096	0.369
μ_ξ	0.266	0.145	1.833
σ_ξ	0.605	0.143	4.240
μ_ξ^Q	-0.043	0.056	-0.764
$\rho_{X,\xi}$	-0.903	0.048	-18.905

Table 1: Model parameters estimated from Copper futures prices, standard deviation (S.D) and t-Test. 2009-2012.

Note that instead of estimating μ_ξ and λ_ξ, we follow Schwartz and Smith (2000) and estimate μ_ξ and μ_ξ^Q with $\mu_\xi = \mu_\xi^Q + \lambda_\xi$, which is equivalent. Thus, the expected return restrictions imposed on λ_ξ are actually reflected in the values of μ_ξ.

In Equation (2), μ_ξ represents the expected long term return on the (log) spot price. In these type of models this parameter is very sensitive to the specific time series considered. In our example its value is 26.6% per year since copper prices where increasing dramatically during the sample period. On the other hand, the risk neutral drift is more influenced by the cross section of futures prices since the futures price formula only contains μ_ξ^Q. In our example its value is -4.3% per year. The implication of this is that, for this extreme example, future expected spot prices will be substantially higher that the futures price for the same maturity.

[5] More details about the estimation will be presented later in the paper.

Using the parameters in Table 1 and the state variables obtained by the Kalman filter for the sample period, the five-year futures and expected five-year spot prices for each date are presented in Figure 1. It can be seen that, for this example, results are totally unreasonable, as it is very unlikely for expected spot prices in five years to be around 5 times the corresponding five-year futures price today, as shown in the figure.

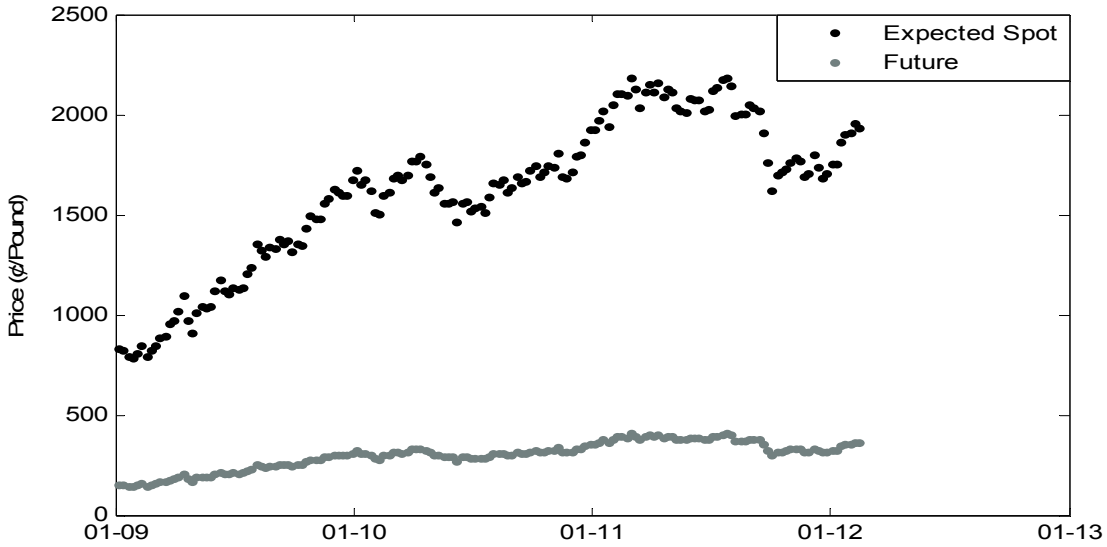

Figure 1: Five year Expected Spot and Futures prices for Copper using the Schwartz and Smith (2000) model.

Given that expected spot and futures prices differ only on the risk premiums, if risk parameters are not well estimated expected spot prices will be unreliable, even though futures prices may not be affected. This fact has been obscured in the literature since most applications of commodity models have been to the pricing of commodity derivatives where only the parameters of the risk neutral process are relevant.

3.- The Expected Futures Returns implied by a Commodity Pricing Model

3.1 Parameter restrictions

All commodity pricing models imply an expected futures return that is, however, never analyzed in the literature. In this section we show how to relate the expected futures returns of this type of models to their parameters. Using the Schwartz-Smith (2000) model presented previously, we derive an equation that

restricts commodity prices to evolve in a consistent way with a given expected futures return. This expression will be used in the next section where we argue that it is useful to use information on futures returns obtained from additional sources to increase commodity pricing model reliability.

We define the expected return between 0 and Δt for a futures contract that matures at time T, $E_0[r_{T,\Delta t}]$, as:

$$E_0[r_{T,\Delta t}] = \frac{E_0[F_{T,\Delta t}]}{F_{T,0}} - 1 \tag{12}$$

To compute the expected value of the futures price at time Δt, defined in Equation (11), we start by defining z, the random (log) futures price at time Δt, as:

$$z = e^{-\kappa(T-\Delta t)}\chi_{\Delta t} + \xi_{\Delta t} + A(T - \Delta \mathrm{t})$$

and compute

$$E_0[F_{T,\Delta t}] = \exp\left(E_0[z] + \frac{1}{2} * var_0[z]\right)$$

with

$$E_0[z] = e^{-\kappa(T-\Delta t)}E_0[\chi_{\Delta t}] + E_0[\xi_{\Delta t}] + A(T - \Delta t)$$

where, from Equation (8),

$$E_0[z] = e^{-\kappa(T-\Delta t)}\left(e^{-\kappa(\Delta t)}\chi_0\right) + \xi_0 + \mu_\xi * \Delta \mathrm{t} + A(T - \Delta t)$$

$$E_0[z] = e^{-\kappa T}\chi_0 + \xi_0 + \mu_\xi * \Delta \mathrm{t} + A(T - \Delta t)$$

and

$$var_0[z] = e^{-2\kappa(T-\Delta t)} var_0[\chi_{\Delta t}] + var_0[\xi_{\Delta t}] + 2e^{-\kappa(T-\Delta t)} cov_0[\chi_{\Delta t}, \xi_{\Delta t}]$$

$$var_0[z] = -e^{-2\kappa T}(1 - e^{2\kappa\Delta t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 \Delta t - 2e^{-\kappa T}(1 - e^{\kappa\Delta t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}$$

Therefore:

$$E_0\left[F_{T,\Delta t}\right] = \exp\left(e^{-\kappa T}\chi_0 + \xi_0 + \mu_\xi * \Delta\mathrm{t} + A(T - \Delta t) + \frac{1}{2} * \left(-e^{-2\kappa T}(1 - e^{2\kappa\Delta t})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 \Delta t - 2e^{-\kappa T}(1 - e^{\kappa\Delta t})\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right)\right)$$

Substituting in (12) and simplifying we obtain the formula for the expected future return over period t:

$$E_0\left[r_{T,\Delta t}\right] = \exp\left(\lambda_\xi \Delta t - e^{-\kappa T}(1 - e^{\kappa\Delta t})\frac{\lambda_\chi}{\kappa}\right) - 1 \qquad (13)$$

Equation (13) is the expected future returns implied by the Schwartz-Smith (2000) model. It can easily be seen that if the risk premium parameters are set to zero, then the expected futures return is also zero, as it should be in the risk neutral world. If information on expected returns for two futures contracts with different maturities were available, we could solve for the two lambdas without resorting to the commodity pricing model which is unable to provide reliable estimates for these parameters, as discussed previously.

In a more general setting the time T futures price at time t, $F_{T,t}$ will be a function of the state variables (X_t) and the model´s parameters (ψ). Regardless of the number of factors considered, Equation (12) will

only be a function of the model parameters, the maturity of the futures contract (T) and the time step considered for the return calculation (Δt).

Thus we can define the right hand side of Equation (13) as $g(\psi, \mathrm{T}, \Delta t)$ for a given commodity pricing model, and restrict the parameter vector ψ to satisfy the following equation:

$$E_0\left[r_{T,\Delta t}\right] = g(\psi, \mathrm{T}, \Delta t) \tag{14}$$

By adding this restriction for any maturity T, one degree of freedom for setting the parameter values is lost. Given that in an N-factor model there are N risk premiums to be estimated, N different expected futures returns for contracts with different maturities are required to estimate all risk premium parameters.

3.2 Parameter estimation

We now show how to estimate the parameters of the Schwartz-Smith (2000) commodity pricing model. We use the Kalman filter and maximum likelihood and include Equation (13) to restrict model expected returns to be equal to specific values described in the next section.

The Kalman filter requires specifying two equations. The first one is the transition equation, which describes the evolution of the state variables for a determined time step Δt:

$$X_t = GX_{t-1} + c + \omega_t$$

From Equation (8):

$$X_t = \begin{bmatrix} \chi_t \\ \xi_t \end{bmatrix}$$

$$c = \begin{bmatrix} 0 \\ \mu_\xi \Delta t \end{bmatrix}$$

$$G = \begin{bmatrix} e^{-\kappa \Delta t} & 0 \\ 0 & 1 \end{bmatrix}$$

and ω_t is a 2×1 vector of serially uncorrelated, normally distributed errors with mean zero and covariance given by Equation (9).

The second equation is the measurement equation, which describes the relationship between the state variables and the observed futures prices:

$$Y_t = J_t X_t + d_t + v_t \tag{15}$$

where[6]

$$Y_t = \begin{bmatrix} \ln\left(F_{t+T_1,t}\right) \\ \vdots \\ \ln\left(F_{t+T_n,t}\right) \end{bmatrix}$$

and, from Equation (11):

$$d_t = \begin{bmatrix} A(T_1) \\ \vdots \\ A(T_n) \end{bmatrix}$$

$$J_t = \begin{bmatrix} e^{-\kappa T_1} & 1 \\ \vdots & \vdots \\ e^{-\kappa T_n} & 1 \end{bmatrix}$$

Also, v_t is a $n \times 1$ vector of serially uncorrelated, normally distributed errors with mean zero and diagonal variance-covariance matrix (R_t).

As X_t and v_t are normally distributed random variables, Y_t is also normally distributed. Thus the probability distribution of Y_t can be determined and the likelihood of the observed futures prices can be computed. This allows estimating the set of parameters by maximum likelihood, but now including one restriction per risk premium parameter, of the type:

$$E_0\left[r_{T,\Delta t}\right] = g(\psi, \mathrm{T}, \Delta t)$$

In addition to including the parameter restrictions derived previously, we estimate the model following Schwartz and Smith (2000) with one important difference. Our data set is much larger and includes a variable number of futures contracts in the cross section. Thus the dimension of the R_t matrix is time varying [Schwartz et al. (2007)] as opposed to constant in Schwartz and Smith (2000). Also, given the much higher dimensionality of our problem, instead of associating a different volatility parameter for each maturity, contracts are classified in five groups according to their maturity and the same volatility parameter

[6] T_1 ... T_n are the maturities of the future contracts.

is associated to each contract within a determined group. Therefore, considering that σ_j is the volatility parameter associated to the j[th] group, R_t has the following structure:

$$R_t = \begin{bmatrix} \overbrace{\sigma_1^2 & 0 & \cdots & 0}^{1^{st}\text{ Group}} & & \overbrace{\cdots}^{j^{th}\text{ Group}} & & \overbrace{\cdots & 0}^{5^{th}\text{ Group}} \\ 0 & \ddots & & & & & & & & \\ \vdots & & \sigma_1^2 & & & & & & & \\ & & & \ddots & & & & & & \\ & & & & \sigma_j^2 & & & & & \\ & & & & & \ddots & & & & \\ & & & & & & \sigma_j^2 & & & \\ & & & & & & & \ddots & & \\ & & & & & & & & \sigma_5^2 & \\ & & & & & & & & & \ddots & 0 \\ 0 & & & & & & & & 0 & \sigma_5^2 \end{bmatrix}$$

4.- Alternative Ways of Estimating Commodity Expected Returns

In the previous sections we have argued that commodity pricing models are not able to provide reliable risk premium parameters and that even though the risk adjusted process may be well estimated, the physical distribution of future spot prices is often very unreliable. We also derived the equation that relates parameter values to model expected returns.

In this section we discuss the different approaches to estimating expected returns on commodity investments without using commodity models. In what follows we present three alternative approaches which have been suggested in the literature, and which could be used to improve the performance of commodity models to estimate the distribution of future spot prices, without detriment to the estimation of futures prices.

4.1 Asset Pricing Models

A number of different asset pricing models have been applied to commodity returns. The starting point of this line of research can be found in Dusak (1973) who studied risk premiums under the Capital Asset Pricing Model (CAPM). Dusak's work focused on three agricultural commodities and found β coefficients close to zero for all of them.

In other related research Bodie and Rosansky (1980) estimate β coefficients for different commodities and find that the CAPM doesn't hold. Carter et al. (1983) discuss the validity of Dusak's selection of the S&P 500 index as the market proxy and state that another index should be used. They also find systematic risk significantly different from zero (for the same contracts studied by Dusak) when β is allowed to be stochastic and it is specified as a function of net market position of large speculators. Chang et al. (1990) finds significant systematic risk for copper, platinum and silver, differing from previous work done on agricultural commodities.

Furthermore, Bessembinder and Chan (1992) and Bjorson and Carter (1997) find that treasury bill yields, equity dividend yields and the 'junk' bond premium have forecasting power in various commodity future markets. Bessembinder (1992) presents results for single and multiple β models[7] while Erb and Harvey (2006) apply a variation of Fama and French (1993) five-factor model to various commodities and commodity portfolios. In both studies no factor is consistently significant across commodities. Bessembinder (1992) also uses his single and multiple β models to test for market integration. He finds no statistical evidence to reject the market integration hypothesis[8] while on a different test finds out that hedging pressure has an impact on commodity and currency futures, but not on financial futures[9].

De Roon et al. (2000) show that hedging pressure on futures contracts and also hedging pressure on other markets (cross-hedging pressures) have significant influence on futures return.

In more recent research Khan et al. (2008) report results for a three-factor model which considers a market proxy, an inventory variable and a hedging pressure variable. The model is applied to copper, crude oil, gold and natural gas presenting mixed results. While the hedging pressure variable holds explanatory power across the four commodities, the other two variables are not statistically significant in all of them.

More recently Hong and Yogo (2010) study the predictability of commodity futures returns. They use a commodity futures portfolio composed of 30 products from the agriculture, energy, livestock and metal

[7] In the single β model the explanatory variable is the return on a market index while in the multiple β model six macroeconomic variables are also considered besides the market index.

[8] This is done by studying the uniformity of risk premiums across assets and futures with an adaptation of the traditional Fama and MacBeth (1973) methodology. He recognizes that the test performed has relatively low power.

[9] The impact of hedging pressure is observed when residual risk, conditional on a hedging pressure variable, is used. This is consistent with Hirshleifer (1988)

sectors. They find that the short rate, the yield spread, the aggregate basis[10] and the open interest growth rate helps to predict commodity futures returns.

Finally Dhume (2010) studies commodity futures returns using a consumption-based asset pricing model developed by Yogo (2006) which extends the classic consumption CAPM (CCAPM) to include durable goods. Dhume finds out that the high correlation between commodities and durable goods consumption growth can explain commodity returns. This finding contrast with Jagannathan (1985) who found that the CCAPM (not including durable goods) was rejected for agricultural commodities.

One of the alternatives for obtaining the expected futures return that will be implemented in the next section is a simple CAPM formulation. Futures contracts are a special case of assets as they represent zero investment positions. Following Chang et al. (1990) and Bessembinder (1992) the CAPM for futures contracts is defined as:

$$E_t(R_{i,T}) = \beta_{i,T}[E_t(R_m) - R_f] \tag{16}$$

where $R_{i,T}$ is the return on the futures price for a contract on the underlying asset i that matures at time $t + T$. Two important details about this specification are worth mentioning. First, for a particular commodity multiple β coefficients can be estimated depending on the time to maturity, T, of the futures contract chosen. Second, this relation implies that the expected return earned by a holder of a long position in the futures contract is only given by the expected risk premium.

When estimating β coefficients from Equation (16) the following regression is run[11]:

$$R_{T,t} = a_T + b_T[R_{m,t} - R_{f,t}] + \epsilon_t \tag{17}$$

where $R_{T,t}$ is the realized return for time period t of a future contract that matures at time $t + T$, $R_{m,t}$ is the realized return on the market portfolio for time period t, $R_{f,t}$ is the risk free rate at time period t, ϵ_t is an

[10] Interesting to note here is that the basis has been found to be related to inventory levels and to the risk premium [Gorton et al., 2013]

[11] For simplicity sub-index i will be dropped from the notation from this point on.

error term and b_T is the estimated value of β_T. Also, if the CAPM holds, a should not be statically different from zero.

Note that to perform the regression a futures contract with exact time to maturity T should be available for each time period ($t \to t + \Delta t$). This is not the case as one futures contract matures each month. Because of this a rolling strategy must be followed in order to hold a contract that has an approximate maturity of T. At the end of each month the futures contract that has the closest time to maturity to the defined value T is selected. This futures contract is held for the next month and by the end of the month the same process is repeated. Once the futures contract is selected, the price of this contract is used to calculate the futures return. Defining $F_{t+T,t}$ as the price at time t of a futures contract that matures at time $t + T$, the return is defined as[12]:

$$E_t(R_T) = E\left(F_{t+T,t+\Delta t}/F_{t+T,t}\right) - 1 \qquad (18)$$

In addition to an estimate of the β_T coefficient, an estimate of the expected market risk premium, $RP = \left[E(R_m) - R_f\right]$, is needed. Damodaran (2009) suggests that there are three alternative approaches to estimate the equity risk premium: (i) survey investors, managers or academics, about their expectations, (ii) use the historical premium (over a certain period of time) as the market expectation and (iii) use implied methods that try to extract the expectations from market prices or rates.

For simplicity the survey approach will be used in the next section. Two types of surveys are available in the literature: those that ask academics (Fernandez (2009), Welch (2001 and 2008)) and those that ask CFO's (Graham and Harvey (2005)). In an unpublished work, Graham and Harvey (2012) update their 2005 work providing quarterly results for the average expected market risk premium since 2000. This is the data set that will be used to compute the commodity futures expected return. The expected return on a futures contract of maturity T is then:

$$E(R_T) = \beta_T \cdot RP \qquad (19)$$

[12] Note that the return is computed for consecutive (separated by a time period of Δt) futures prices that mature at the same date ($t + T$).

To best estimate β_T we use a dynamic approach[13] in which coefficients are calculated for every time instant t using two-years back looking rolling windows. Thus a time series of coefficients are obtained.

4.2 Zero Risk Premium

Given the difficulty of estimating the risk premium and its time varying nature, some authors[14] and practitioners assume that the commodity futures risk premium is zero. This implies that the true and the risk neutral distributions are the same.

4.3 Expert Opinion

Many commodity producers and investment banks regularly provide estimates of future expected spot prices, and therefore expected commodity returns, using proprietary models based on supply and demand estimations, technological developments and political uncertainty.

5.- Results

In this section we implement our framework requiring model returns to match CAPM and Zero risk premium estimates. Expert opinion values, when available, could easily be incorporated into our framework.

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[13] In addition we also used a static approach in which a single β_T coefficient is estimated using return data from the same time window considered for the model calibration. Results (not reported) were very similar with those using the dynamic approach.

[14] For example, Fama and French (1987) state in their conclusion: "Likewise, the large variances of realized premiums mean that average premiums that often seem economically large are usually insufficient to infer that expected premiums are nonzero, especially in the data for individual commodities."

5.1 Data

The model was estimated for two commodities: copper and oil. The data used in the estimation can be divided into three parts: (i) Commodity futures prices, (ii) Market information and (iii) Market Surveys.

Regarding commodity futures, copper data was obtained from the Commodity Exchange, Inc (COMEX) and oil data from the New York Mercantile Exchange (NYMEX). Copper data was complemented with London Metal Exchange (LME) long term contracts[15]. Weekly (Tuesday closing) futures prices contracts from January 1995 until December 2012 were used. For copper, the number of contracts traded each date ranged from 12 to 40, while for oil between 12 and 78[16]. Figures 2 and 3 show a time series of futures term structures for each commodity. As can be seen from the figures the shape and level of the futures curves, and the number of traded contracts varied significantly during the sample period.

Market information consists of a time series of weekly closing prices for the Standard & Poor's 500 Index (S&P 500) and for the three-month Treasury bill rate. These were used as proxies for the equity market and for the risk free rate necessary for estimating the futures risk premiums.

[15] One or two contracts with maturities at least one year over the longest COMEX contract were added.

[16] Before February 2006 the number of contracts available at a single date was rarely more than 35. Since February 2006 contracts available in the data set went to more than 70. Given the high number of contracts for each date from February 2006, a sample of contracts was selected. The selection always considered the first five futures and then one in every two contracts were also selected, making sure that the longest maturity contract was always in the estimation set.

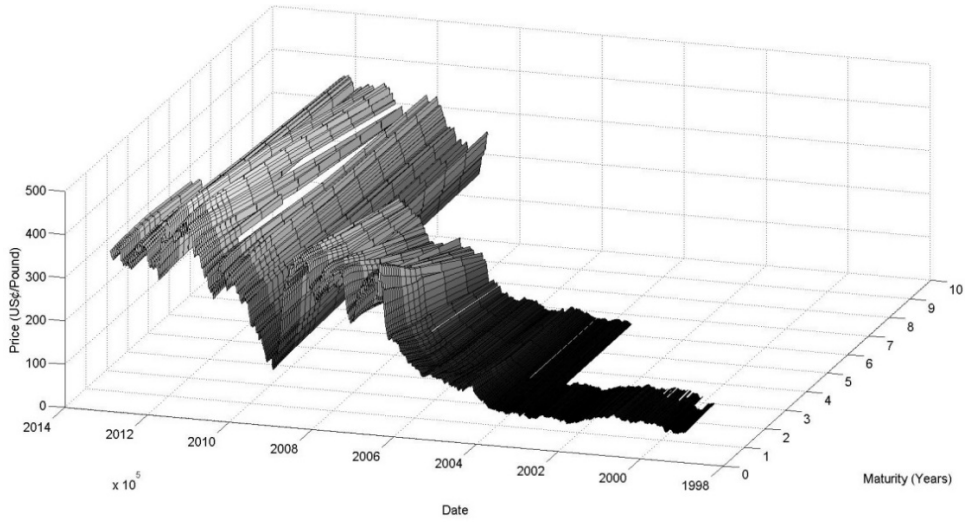

Figure 2: Copper (COMEX) Futures Term Structure

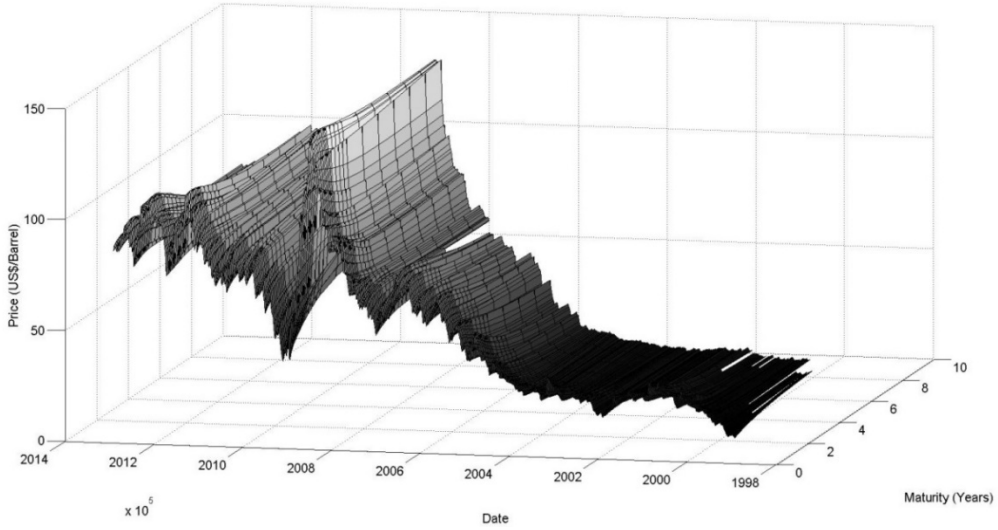

Figure 3: WTI Oil (NYMEX) Futures Term Structure

Finally the survey information on expected market risk premiums was obtained from Graham and Harvey (2012). Figure 4 presents the quarterly surveys results on the expected market risk premium from Chief Financial Officers (CFOs) for the period June 2000 to March 2012[17]. Weekly expected equity risk premiums are obtained by linear interpolation.

17 The exact question asked to CFOs was about the average expected market return over the next 10 years.

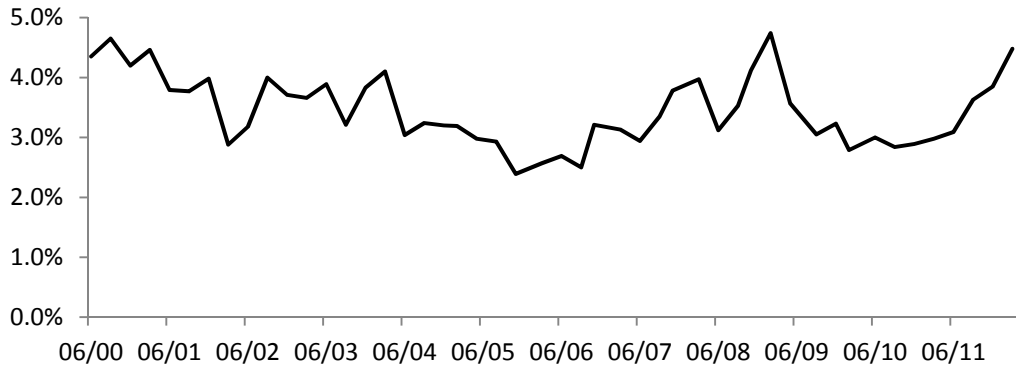

Figure 4: Expected market risk premium from Graham and Harvey (2012). Data is obtained from surveys to CFO's.

5.2 Kalman Filter Parameter Estimations

The model was estimated for two five-year[18] windows (2001-2006 and 2006-2011) and one additional three-year window (January 2009 to February 2012) that does not include the financial crisis. Data between February and December 2012 was used for out-of-sample tests.

Tables 2 to 7 show copper and oil models' parameters for each time window. In every table, results for the Asset Pricing (CAPM), Zero and Non-Restricted risk premium parameter estimations are shown[19]. The first two parameter estimations correspond to restricting the model to generate expected futures returns equal to those from the Asset Pricing model or to zero. The non-restricted parameter estimation shows the result of using only information from future contracts to estimate the model, as it has traditionally been done in the commodity pricing literature.

As explained earlier, instead of reporting μ_ξ and λ_ξ, we follow Schwartz and Smith (2000) and estimate μ_ξ and μ_ξ^Q with $\mu_\xi = \mu_\xi^Q + \lambda_\xi$.

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18 The actual length is 5 years and one month as it was the case in Schwartz and Smith (2000)

19 The results for μ_ξ and λ_χ in the Asset Pricing restricted case are time varying because they depend on the other parameters (which are constant) but also on the expected returns which are time varying as a consequence of the time variation in the expected market risk premium information and estimated β coefficient. The results presented in the tables correspond to the value for the last time instant of each window.

It can be observed from the tables that estimates for μ_ξ and λ_χ are different between the non-restricted and the restricted cases. In contrast, the impact of this restriction on the other parameters is much smaller.

Parameter	Asset Pricing			Zero			Non-Restricted		
	Estimate	S.D	T-Test	Estimate	S.D	T-Test	Estimate	S.D	T-Test
κ	0.475	0.006	80.868	0.475	0.006	80.869	0.475	0.006	80.867
σ_X	0.218	0.010	22.712	0.218	0.010	22.713	0.218	0.010	22.719
λ_X	0.005	--	--	0.000	--	--	-0.006	0.028	-0.213
μ_ε	-0.024	--	--	-0.026	--	--	0.194	0.083	2.325
σ_ε	0.205	0.009	22.129	0.205	0.009	22.135	0.204	0.009	22.057
μ_ε^Q	-0.026	0.002	-12.832	-0.026	0.002	-12.826	-0.026	0.002	-12.825
$\rho_{X,\varepsilon}$	-0.393	0.054	-7.247	-0.391	0.054	-7.217	-0.405	0.054	-7.553

Table 2: Model parameters estimated from Copper futures prices, standard deviation (S.D) and t-Test. 2001-2006

Parameter	Asset Pricing Model			Zero			Non-Restricted		
	Estimate	S.D	t-Test	Estimate	S.D	t-Test	Estimate	S.D	t-Test
κ	0.084	0.005	16.507	0.080	0.008	9.873	0.103	0.004	26.410
σ_X	1.313	0.054	24.419	1.360	0.094	14.438	1.152	0.027	43.405
λ_X	0.008	--	--	0.000	--	--	0.233	0.049	4.749
μ_ξ	-0.189	--	--	-0.215	--	--	0.178	0.143	1.246
σ_ξ	1.118	0.053	21.170	1.172	0.090	12.964	0.795	0.044	18.107
μ_ξ^Q	-0.202	0.017	-11.957	-0.215	0.018	-11.835	-0.037	0.025	-1.505
$\rho_{X,\xi}$	-0.958	0.005	-189.468	-0.962	0.006	-149.319	-0.915	0.014	-67.362

Table 3: Model parameters estimated from Copper futures prices, standard deviation (S.D) and t-Test. 2006-2011

Parameter	Asset Pricing Model			Zero			Non-Restricted		
	Estimate	S.D	t-Test	Estimate	S.D	t-Test	Estimate	S.D	t-Test
κ	0.110	0.009	12.301	0.110	0.009	11.883	0.111	0.012	9.513
σ_X	0.912	0.054	17.004	0.912	0.055	16.433	0.910	0.069	13.180
λ_X	0.012	--	--	0.000	--	--	0.036	0.096	0.369
μ_ξ	-0.031	--	--	-0.064	--	--	0.266	0.145	1.833
σ_ξ	0.627	0.052	12.173	0.640	0.054	11.945	0.605	0.143	4.240
μ_ξ^Q	-0.056	0.010	-5.801	-0.064	0.010	-6.441	-0.043	0.056	-0.764
$\rho_{X,\xi}$	-0.910	0.017	-52.579	-0.912	0.017	-53.175	-0.903	0.048	-18.905

Table 4: Model parameters estimated from Copper futures prices, standard deviation (S.D) and t-Test. 2009-2012

Parameter	Asset Pricing Model			Zero			Non-Restricted		
	Estimate	S.D	t-Test	Estimate	S.D	t-Test	Estimate	S.D	t-Test
κ	1.216	0.010	119.4	1.216	0.010	119.420	1.216	0.010	119.410
σ_X	0.726	0.010	73.364	0.726	0.010	73.390	0.726	0.010	73.384
λ_X	-0.014	--	--	0.000	--	--	0.233	0.156	1.489
μ_ξ	-0.036	--	--	-0.030	--	--	0.208	0.083	2.503
σ_ξ	0.200	0.012	17.054	0.200	0.012	17.171	0.197	0.011	17.224
μ_ξ^Q	-0.030	0.002	-12.352	-0.030	0.002	-12.440	-0.029	0.002	-12.591
$\rho_{X,\xi}$	0.360	0.113	3.181	0.353	0.114	3.094	0.341	0.116	2.943

Table 5: Model parameters estimated from Oil futures prices, standard deviation (S.D) and t-Test. 2001-2006

Parameter	Asset Pricing Model			Zero			Non-Restricted		
	Estimate	S.D	t-Test	Estimate	S.D	t-Test	Estimate	S.D	t-Test
κ	0.278	0.004	78.431	0.277	0.004	77.779	0.277	0.004	77.790
σ_X	0.551	0.005	112.099	0.551	0.005	111.625	0.551	0.005	111.762
λ_X	0.006	--	--	0.000	--	--	0.163	0.045	3.629
μ_ξ	-0.001	--	--	-0.017	--	--	0.081	0.113	0.719
σ_ξ	0.310	0.018	16.771	0.291	0.017	17.091	0.276	0.015	18.436
μ_ξ^Q	-0.022	0.006	-3.938	-0.017	0.005	-3.390	-0.012	0.004	-2.992
$\rho_{X,\xi}$	-0.536	0.060	-8.894	-0.477	0.069	-6.928	-0.384	0.085	-4.546

Table 6: Model parameters estimated from Oil futures prices, standard deviation (S.D) and t-Test. 2006-2011

Parameter	Asset Pricing Model			Zero			Non-Restricted		
	Estimate	S.D	t-Test	Estimate	S.D	t-Test	Estimate	S.D	t-Test
κ	0.414	0.004	112.174	0.414	0.004	112.119	0.414	0.004	112.403
σ_X	0.578	0.005	124.460	0.578	0.005	124.402	0.579	0.005	124.744
λ_X	0.018	--	--	0.000	--	--	0.220	0.029	7.490
μ_ξ	0.021	--	--	-0.010	--	--	0.036	0.119	0.305
σ_ξ	0.253	0.020	12.585	0.255	0.020	12.435	0.216	0.014	15.335
μ_ξ^Q	-0.010	0.005	-1.921	-0.010	0.005	-1.953	-0.001	0.003	-0.333
$\rho_{X,\xi}$	-0.442	0.080	-5.534	-0.441	0.080	-5.489	0.240	0.145	1.652

Table 7: Model parameters estimated from Oil futures prices, standard deviation (S.D) and t-Test. 2009-2012

5.3 Model Fit

We now analyze the impact of the proposed approach on model fit. Tables 8 and 9 show the in-sample and out-of-sample mean absolute errors between model and market futures prices for copper and oil, for each of the parameter estimation approaches and time windows. The errors are presented as percentage of the observed futures price.

Regarding futures prices in-sample fit, the three methodologies give the same good performance. In fact, considering both commodities, the mean absolute error is less than 1.5% for all time windows. Moving to the out-of-sample fit, the mean absolute error for each time window is in general larger than for the in-sample test, but still errors for copper are always less than 2.5% and for oil less than 1.5% and basically the same regardless of the estimation methodology.

These results show that restricting parameter values to match the expected return obtained by other methods has no significant effect in pricing futures.

Window	In Sample			Out of Sample		
	Asset Pricing	Zero	Non-Restricted	Asset Pricing	Zero	Non-Restricted
2001-2006	0.4%	0.4%	0.4%	2.2%	2.2%	2.2%
2006-2011	0.8%	0.7%	0.7%	0.9%	0.9%	0.9%
2009-2012	0.7%	0.7%	0.7%	0.5%	0.5%	0.5%

Table 8: Copper In-Sample and Out-of-Sample Mean Absolute Error for the three methods. Errors are calculated as percentage of the observed futures price.

Window	In Sample			Out of Sample		
	Asset Pricing	Zero	Non-Restricted	Asset Pricing	Zero	Non-Restricted
2001-2006	0.7%	0.9%	0.9%	1.3%	1.3%	1.3%
2006-2011	1.5%	1.3%	1.3%	0.8%	0.8%	0.8%
2009-2012	0.8%	0.8%	0.8%	1.4%	1.4%	1.4%

Table 9: Oil In-Sample and Out-of-Sample Mean Absolute Error for the three methods. Errors are calculated as percentage of the observed futures price.

We now study the effect of the above restrictions on expected spot prices ($E_t[S_T]$). Figures 5 and 6 show an example of a futures term structure and the corresponding expected spot prices for copper and oil, respectively, for one particular date. It can be seen that expected spot prices for the zero risk premium assumption is the same as the futures curve, as it should be. But we can also see that expected spot prices when using the asset pricing model for estimating risk premiums are much different from those from the non-restricted case.

Figures 7 and 8 show the five year futures price and the expected 5-year spot price for both copper and oil over the last three years of the sample period. They are equivalent to Figure 1, only that now the restricted expected spot prices are also included. Results for the restricted estimations seem clearly more reasonable than those from the non-restricted case. Our results are consistent with those of Heath (2013) who reports that different risk premium specifications have an equivalent performance in fitting futures contracts, but provide considerably different price forecasts.

Therefore, restricting some parameter values such that expected futures returns are consistent with those of an asset pricing model has the positive consequence of providing an expected spot price that incorporates new information in the estimation of the risk premiums, gives an expected spot price that is more reasonable and achieves this without losing the ability to adequately price futures contracts.

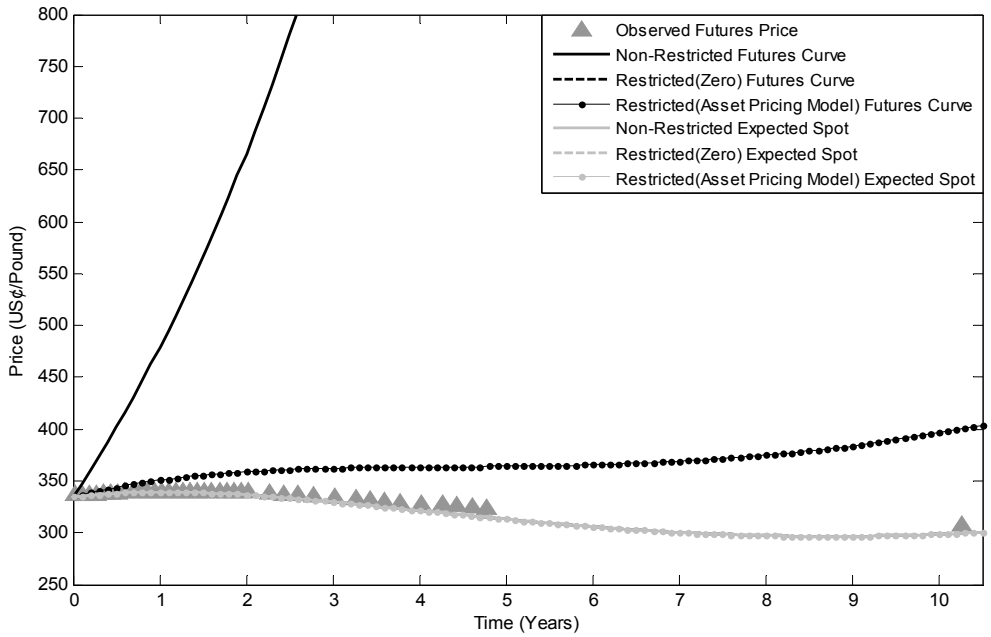

Figure 5: Copper Futures (grey) and Expected Spot Prices (black) for 12-20-2011. (2009-2012 parameters).

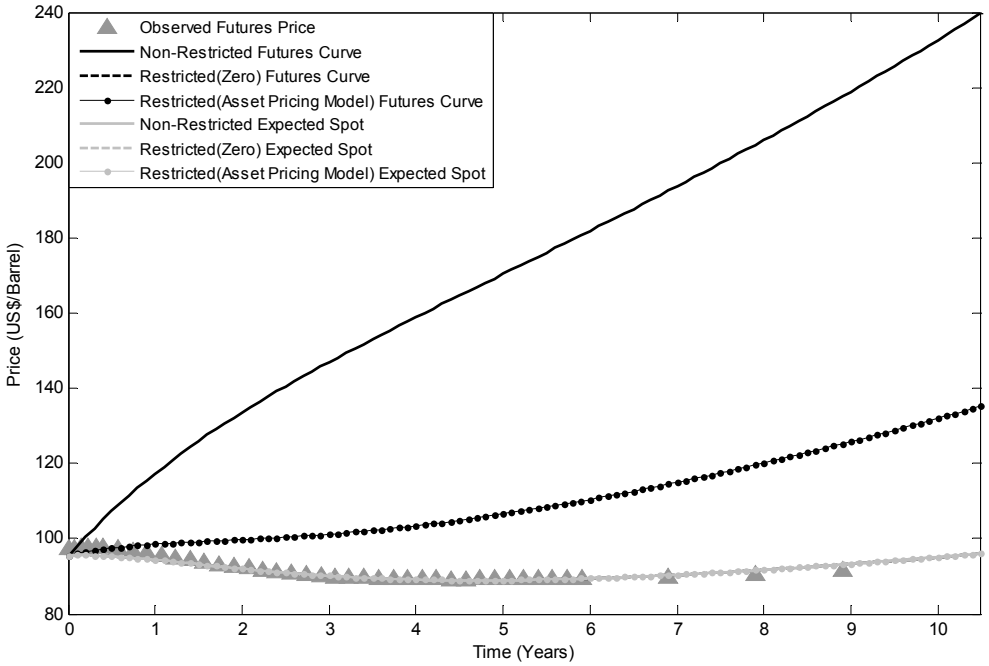

Figure 6: Oil Futures (grey) and Expected Spot Prices (black) for 12-20-2011. (2009-2012 parameters)

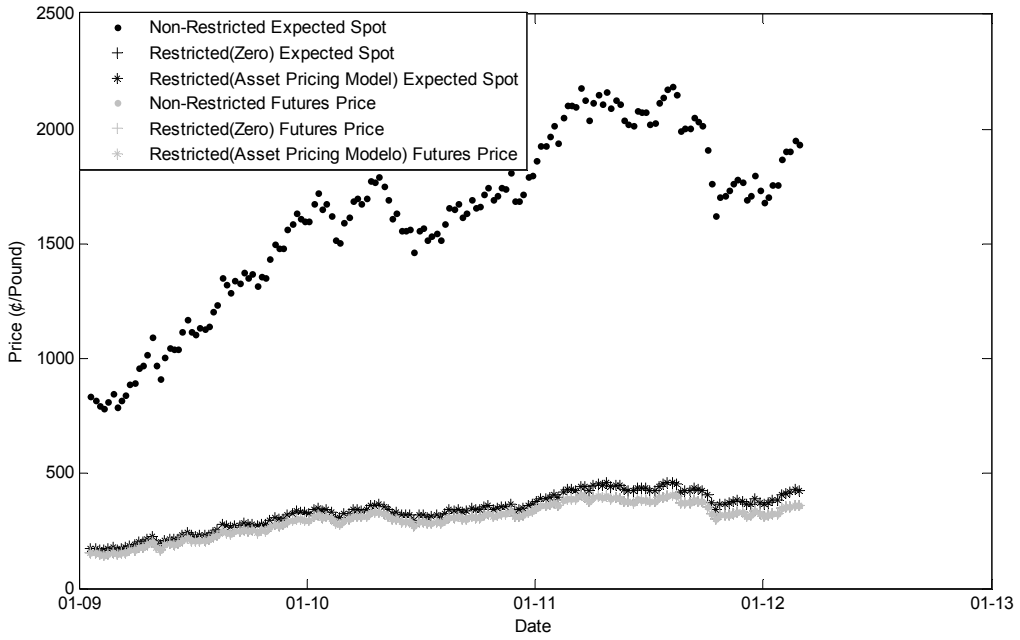

Figure 7: Five-year Copper Futures (grey) and Expected Spot Prices (black) for 2009-2012. (2009-2012 parameters)

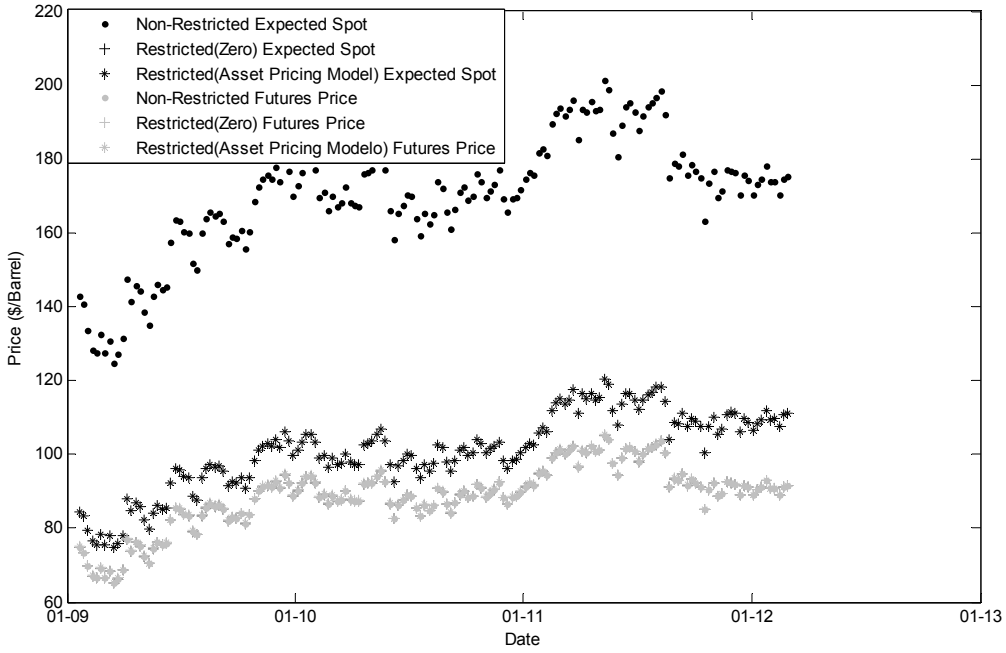

Figure 8: Five-year Oil Futures (grey) and Expected Spot Prices (black) for 2009-2012. (2009-2012 parameters)

6.- Conclusion

We present a simple methodology to improve the performance of commodity price models. Given current evidence on the financialization of commodity markets it seems reasonable to require commodity models to behave more like an investment asset and use this information to improve the performance of the commodity models. This approach provides a more robust estimation of risk premium parameters and therefore more credible expected future spot prices, without compromising the model´s fit to futures contracts price observations. We explore this approach by using asset pricing models, in particular a CAPM version, and also assuming a zero expected futures return.

The procedure first defines the expected futures return implicit in the commodity pricing model as a function of the model's parameters. Then, it imposes a set of restrictions on the parameter estimation process so that these expected futures returns are consistent with those from alternative sources. In this way new information, not available in traditional estimation of commodity models, is included which gives not only an excellent fit to observed futures prices, but also provides reasonable expectations for future spot prices.

To illustrate the methodology we use the Schwartz and Smith (2000) commodity pricing model. We estimate the model for copper and oil futures contracts, considering different time windows between 2001 and 2012. We restrict the commodity pricing process to match the expected returns obtained from the CAPM and also zero expected returns. For comparison, we also estimate the commodity model ignoring alternative information, as is traditional in the literature.

Our results show that the methodology has an important benefit relative to the traditional estimation because the expected spot prices implied by the restricted model are much more reasonable and consistent with other models and beliefs. This improvement is obtained with no significant difference in the model's ability to fit future contracts prices, offering equivalent measures of in-sample and out-of-sample mean absolute error.

The proposed methodology makes commodity pricing models more credible and useful for many practical applications, and may help expand their use among practitioners.

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